TEXAS ALLIANCE FOR MINORITIES IN ENGINEERING
STATE MATH & SCIENCE COMPETITION

Calculus Test

General Information:

- Check to see that you have written your name and Social Security Number on the Scantron sheet and bubbled them in correctly.
- DO NOT open the test booklet and DO NOT start until the proctor says "begin."
- Each individual exam period will be 45 minutes and each exam contains 50 multiple choice questions.
- Students are allowed to use a pre-approved battery operated calculator during the individual and team exams.
- Students are encouraged to write on the exam booklet. Scratch paper and pencils will also be provided. All materials (exam booklet, scratch paper, pencils, and scantron form) are to be turned in when the proctor tells you to. No materials are to leave the room.
- Students will not be permitted to leave the test room while the test is in progress. If a student finishes early, he/she must remain in the test room until the exam period is completed.
- If you need to ask a question during the test, raise your hand and the proctor will come to you.
- There is no penalty for skipping a problem. The exam scores will be determined by the number of correct answers. All ties will be broken by awarding the place to the contestant who has the most consecutive correct answers before a problem is missed.
- **Students may NOT keep the test booklet.**

Calculus Test

x denotes the natural logarithm of x (i.e., log to the base e)
The domain of function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_{-2}^{-1} x^{-2}dx =$

   a. $-\frac{1}{2}$

   b. $\frac{7}{16}$

   c. $\frac{7}{4}$

   d. $-\frac{7}{4}$

   e. $\frac{1}{2}$

2. If $f(x) = \pi^2$, then $f'(1) =$

   a. 2π

   b. 0

   c. π

   d. 1

   e. π^2

3. If $y = \frac{1}{\sqrt[3]{e^x}}$, then $\frac{dy}{dx} =$

   a. $\frac{1}{3\sqrt[3]{e^x}}$

   b. $\frac{3}{\sqrt[3]{e^x}}$

   c. $\frac{-1}{3\sqrt[3]{e^x}}$

   d. $\frac{-(e^x)^{\frac{-4}{3}}}{3}$

   e. $(\ln \sqrt{e^x}) \cdot \frac{1}{3}(e^x)^{-2/3}$

4. $\lim_{h \to 0} \frac{\sin(\Pi + h) - \sin \Pi}{h}$

   a. 1

   b. 0

   c. -1

   d. $+\infty$

   e. $-\infty$

5. The slope of the line tangent to the curve $y^3 + x^2y^2 - 3x^3 = 9$ at (1,2) is

a. $\frac{1}{16}$

b. $\frac{7}{16}$

c. $-\frac{25}{16}$

d. $-\frac{5}{16}$

e. $-\frac{7}{16}$

6. If $f'(x) = \sin x$ and $f(\pi) = 3$, then $f(x) =$

a. $\cos x + 4$

b. $-\cos x + 2$

c. $-\cos x + 4$

d. $\cos x + 3$

e. $-\cos x - 2$

7. The position of a particle moving along a straight line at any time t is given by $s(t) = 2t^3 - 4t^2 + 2t - 1$. What is the acceleration of the particle when $t = 2$?

a. 32

b. 16

c. 4

d. 8

e. 0

8. If $f[g(x)] = \sec(x^3+4)$, $f(x) = \sec x^3$, and $g(x)$ is not an integer multiple of $\frac{\pi}{2}$, then $g(x)$

a. $\sqrt[3]{x+4}$

b. $\sqrt[3]{x-4}$

c. $\sqrt[3]{x^3+4}$

d. $\sqrt[3]{x} - 4$

e. $\sqrt[3]{x} + 4$

9. The equation of each horizontal asymptote for $f(x) = \frac{1-|x|}{x}$

a. $y = 1$

b. $y = -1$

c. $x = 0, x = 1, x = -1$

d. $y = 0$

e. $y = 1, y = -1$

10. The acceleration of a particle moving on a line is $a = t^{-1/2} + 3t^{1/2}$. What velocity did the particle have from $t = 0$ to $t = 9$?

a. 60

b. 63

c. -1

d. 1

e. $\frac{93}{2}$

11. The domain of the function defined by $f(x) = \ln(x^2 - x - 6)$ is the set of all real numbers such that

a. $x > 0$

b. $-2 \le x \le 3$

c. $-2 \le x$ or $x \ge 3$

d. $-2 < x < 3$

e. $-2 < x$ or $x > 3$

12. $\int_1^e \frac{\ln(x^2)}{x} dx =$

a. 2

b. $e^2 - 1$

c. $\frac{1}{3}$

d. 1

e. $\frac{1}{3}(e^3 - 1)$

13. If $y \arccos(\cos^4 x - \sin^4 x)$, then $y'' =$

a. 2

b. 0

c. $-2(\cos x - \sin x)$

d. $-2(\sin x + \cos x)$

e. -1

14. If $\frac{f(x_1)}{f(x_2)} = f\left(\frac{x_1}{x_2}\right)$ for all real numbers x_1 and x_2, where $x \neq 0$ and $f(x) \neq 0$, which of the following could define f?

a. $f(x) = \frac{1}{x}$

b. $f(x) = x^2 + 3$

c. $f(x) = x + 1$

d. $f(x) = \ln x$

e. $f(x) = e^x$

15. $\lim_{x \to 1} \frac{\ln(x^3 e^x)}{x} =$

a. $\frac{3(\ln x + e^x)}{x}$

b. $\ln(x^3 e^x - x)$

c. $\ln x^2 + 1$

d. $\frac{3 \ln x + x}{x}$

e. $\frac{3 \ln x}{x}$

16. $\lim_{x \to 1} \frac{\frac{1}{x+1} - \frac{1}{2}}{x - 1} =$

a. $-\frac{1}{4}$

b. -1

c. $\frac{1}{4}$

d. 0

e. does not exist

17. If $\frac{r^2}{r-1}$ r, then

a. $r \geq 0$

b. $r \leq 0$

c. $r \leq 0$ or $r > 1$

d. $r \leq 0$ or r 1

e. $0 \leq r < 1$

18. If $f'(c) = 0$ for $f(x) = 3x^2 - 12x + 9$, where $0 \leq x \leq 4$, then $c =$

a. 2

b. 3

c. 0

d. 1

e. $\frac{1}{3}$

19. $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} =$

a. 6

b. -6

c. 0

d. -12

e. $+\infty$

20. $\int \left(x - \frac{1}{x}\right)^2 dx =$

a. $\frac{1}{3}\left(x - \frac{1}{x}\right)^3 + c$

b. $\frac{1}{3}\left(x - \frac{1}{x}\right)^3 \left(1 + \frac{1}{x^2}\right) + c$

c. $\frac{1}{3}x^3 - 2x - \frac{1}{x^2} + c$

d. $\frac{1}{3}x^3 - 2x - \frac{1}{x} + c$

e. $\frac{1}{3}(1 - \ln x)^3 + c$

21. If $e^{g(x)} = \frac{x^x}{x^2 - 1}$, then $g(x) =$

a. $x \ln x - 2x$

b. $\frac{\ln x}{2}$

c. $(x - 2) \ln x$

d. $\frac{x \ln x}{\ln(x^2 - 1)}$

e. $x \ln x - \ln (x^2 - 1)$

22. If $h(x) = \frac{x^2 + 1}{x^2}$ where $x > 1$, then $h^{-1}(x) =$

a. $\frac{1}{\sqrt{x-1}}$

b. $\sqrt{\frac{x}{1+2x}}$

c. $\frac{1}{\sqrt{x}}$

d. $\frac{1}{\sqrt{x-1}+1}$

e. $\frac{1}{-\sqrt{x-1}}$

23. If $f(x) = \begin{cases} \dfrac{2x-6}{x-3} & x \neq 3 \\ 5 & x = 3 \end{cases}$, then $\lim_{x \to 3} f(x) =$

a. 5
b. 1
c. 2
d. 6
e. 0

24. If $f(x) = \dfrac{\sqrt{x+2}}{x+2}$ and $g(x) = \dfrac{1}{x}$-2, then $f[g(x)] =$

a. $\dfrac{\sqrt{\dfrac{1}{x}-2}}{\dfrac{1}{x}-2}$

b. $\sqrt{\dfrac{1-2x}{x}}$

c. $\dfrac{\sqrt{\dfrac{1}{x-2}+2}}{\dfrac{1}{x-2}+2}$

d. $\sqrt{x}$

e. $\dfrac{\sqrt{x}}{x}$

25. If $\tan x = 2$, then $\sin 2x =$

a. $\dfrac{2}{5}$

b. $\dfrac{4\sqrt{5}}{5}$

c. $\dfrac{4}{5}$

d. $\dfrac{4}{3}$

e. $\dfrac{2}{3}$

26. If $f(x) = \log_b x$ then $f(bx) =$

a. $bf(x)$
b. $f(b)\, f(x)$
c. $1 + f(x)$
d. $xf(b)$
e. $f(x)$

27. IF $f(x) = \begin{cases} x+1 & x \leq 1 \\ 3+ax^2 & x > 1 \end{cases}$ then $f(x)$ is continuous for a =

a. 1
b. -1

c. $\frac{1}{2}$

d. 0

e. -2

28. If $g(x) = \frac{-x - f(x)}{f(x)}$, $f(1)=4$ and $f'(1)=2$, then $g'(1)=$

a. $-\frac{1}{2}$

b. $\frac{11}{8}$

c. $\frac{3}{16}$

d. $\frac{1}{8}$

e. $-\frac{1}{8}$

29. The domain of $f(x) = \sqrt{4 - x^2}$ is

a. $-2 \le x \le 2$

b. $-2 \le x$ or $x \ge 2$

c. $-2 < x$ or $x > 2$

d. $-2 < x < 2$

e. $x \ge 2$

30. $\int \frac{x + e^x}{xe^x} dx =$

a. $-e^{-x} - \frac{1}{x^2} + c$

b. $e^{-x} - \ln|x| + c$

c. $-e^{-x} - \ln|x| + c$

d. $-\frac{1}{e^{2x}} + \ln|x| + c$

e. $e^{-x} - \frac{1}{x^2} + c$

31. The area enclosed by the graphs of $y = x^2$ and $y = 2x+3$ is:

a. $\frac{38}{3}$

b. $\frac{40}{3}$

c. $\frac{34}{3}$

d. $\frac{16}{3}$

e. $\frac{32}{3}$

32. The volume of revolution formed by rotating the region bounded by $y = x^3$, $y = x$, $x=0$ and $x=1$ about the x-axis is represented by

a. $\pi\int_0^1 (x^3 - x)^2\,dx$

b. $\pi\int_0^1 (x^6 - x^2)dx$

c. $2\pi\int_0^1 (x^2 - x^6)dx$

d. $\pi\int_0^1 (x^2 - x^6)dx$

e. $2\pi\int_0^1 (x^6 - x^2)dx$

33. The vertical asymptote and horizontal asymptote for $f(x) = \dfrac{\sqrt{x}}{x+4}$ is

a. $x = -4$, $y = 0$

b. no vertical asymptote, $y = 0$

c. no vertical or horizontal asymptote

d. $x = -4$, no horizontal asymptote

e. $x = -4$, $y = 1$

34. If $f(x) = x^3 - x$, then

a. $\dfrac{\sqrt{3}}{3} = x$ is a local maximum of f

b. $\dfrac{\sqrt{3}}{3} = x$ is a local minimum of f

c. $\sqrt{3} = x$ is a local maximum of f

d. $\sqrt{3} = x$ is a local minimum of f

e. $-\sqrt{3} = x$ is a local minimum of f

35. If $\int_a^b f(x)dx = 0$ then

a. $f(x) = 0$

b. $a = b$

c. $f(x) = 0$ or $a = b$

d. $f(-x) = -f(x)$

e. None of these

36. $\dfrac{d}{dx}\left(\int_0^3 \dfrac{1}{1+x}dx\right) =$

a. $-\dfrac{3}{4}$

b. $\ln 4$

c. 0

d. $\dfrac{1}{4}$

e. $\dfrac{3}{4}$

37. $\int_{-3}^{2} dx =$

a. 5

b. -1

c. 0

d. undefined

e. -5

38. If $h(x) = f[g(x)]$ with $g(x) = x^2 + 4x$, $f'(12) = 2$, $f''(8) = 3$ and $f'(2) = 4$, then $h'(2) =$

a. 32

b. 24

c. 48

d. 16

e. 36

39. If $y = \sqrt[4]{(1-4x)^3}$, then $y'' =$

a. $\dfrac{-3}{\sqrt[4]{(1-4x)^5}}$

b. $\dfrac{-3}{\sqrt[5]{(1-4x)^4}}$

c. $\dfrac{3}{\sqrt[4]{(1-4x)^5}}$

d. $15(1-4x)^{-9/4}$

e. $3(1-4x)^{-1/4}$

40. If $\int_a^b cxdx = a^2 - b^2$ and a ≠ b, then c =

a. -2

b. 2

c. $\dfrac{1}{2}$

d. -x

e. $-\dfrac{1}{2}$

41. If $f(x) = x^3 - \dfrac{1}{x}$, then $f'''(x) =$

a. $-\dfrac{6}{x^4}$

b. $\dfrac{6}{x^4}$

c. $6 + \dfrac{6}{x^4}$

d. $6 - \dfrac{6}{x^4}$

e. $6x + \dfrac{6}{x^4}$

42. The average value of $f(x) = \sqrt{9-x}$ on the interval $0 \le x \le 9$ is

a. 6

b. 2

c. $1\dfrac{1}{2}$

d. -6

e. -2

43. If $8x - 11 \le f(x) \le 2x^2 - 3$ for all x on the interval $0 \le x \le 4$, then $\lim_{x \to 2} f(x)$ is

a. -11

b. -3

c. 29

d. 21

e. 5

44. $\left(\dfrac{\tan x - \cot x}{\sec x(\csc x)}\right)' =$

a. $-4 \cos x \sin x$

b. 1

c. $4 \sin x \cos x$

d. $\dfrac{\sin x - \cos x}{\cos^2 x \sin^2 x}$

e. $\left(\dfrac{\cos x - \sin x}{\cos^2 x \sin^2 x}\right)'$

For questions 45, 46, 47: an object moves in a straight line for 8 seconds with $x(t) = 8t^2 - \left(\dfrac{2}{3}\right)t^3$, t in [0, 8]

45. How far does the object go?

a. $\frac{512}{3}$

b. $\frac{128}{3}$

c. $\frac{1024}{9}$

d. $\frac{244}{9}$

e. $\frac{256}{3}$

46. What is its maximum velocity?

a. 24 m/s

b. 18 m/s

c. 32 m/s

d. 48 m/s

e. 12 m/s

47. How far has the object gone when it reaches its maximum velocity?

a. $\frac{512}{3}$

b. $\frac{128}{3}$

c. $\frac{1024}{9}$

d. $\frac{244}{9}$

e. $\frac{256}{3}$

For questions 48, 49, & 50: if $f(x) = \frac{1-x^2}{x^2+1}$, then

48. Find the $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

a. 2

b. -2

c. 1

d. -1

e. 0

49. Find the intervals where f increases.

a. $x > 0$

b. $x < 0$

c. $x > 2$

d. $x < 2$

e. $x > 1$

50. Find the equation of the tangent line that is parallel to the x-axis.

a. $y = 2x$

b. $y = x^2$

c. $y = 1$

d. $3y = x$

e. $y = 4$

Calculus Test
Answer Key

1. e
2. b
3. c
4. c
5. a
6. b
7. b
8. c
9. e
10. a
11. e
12. d
13. b
14. a
15. d
16. a
17. c
18. a
19. b
20. d
21. e
22. a
23. c
24. d
25. c
26. c
27. b
28. e
29. a
30. c
31. e
32. d
33. b
34. b
35. e
36. c
37. a
38. d
39. a
40. a
41. c
42. b
43. e
44. c
45. a
46. c
47. e
48. d
49. b
50. c